A deep learning based multigrid multiscale method for solving the Navier-Stokes-equations

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The work introduces a novel method for machine learning enhanced simulations. The objective is to improve coarse-grid solutions of the Navier-Stokes-equations in a Multigrid framework. Using a geometric Multigrid method for solving the Navier-Stokes-equations, a hierarchy of meshes is available. The aim is to predict the defect of a coarse grid solution w. r. t. a fine grid solution. The prediction is made using a neural network, outputting a correction on a refined mesh without solving on this level. To gain some domain independence, we follow a local approach: every coarse grid cell makes up a patch of fine cells. The machine learning model takes local information of the coarse cell and outputs a defect on the refined coarse cell as depicted in figure 1.

We aim at enhancing not only one solution but the whole process by incorporating the output ouf the network into the time evolution. Since time discretization is done with a Crank-Nicholson method, this is fairly straightforward. The solution from the old timestep is augmented by the restriction of the enhanced solution onto the coarse grid.

The idea of modelling a defect between resolved scales (coarse grid solution) and unresolved scales (defect w. r. t. fine grid solution) also takes a strong resemblance to variational multiscale methods (VMS). With this the method is built on top of well-established mathematical models and tries to combine their strengths with those of artificial neural networks (ANN). ANNs are mostly known for their capability of learning complex, nonlinear relations and temporal dependencies, which motivates their use in fluid dynamics and the proposed method in particular.

As test case we apply the method to a channel flow problem with an obstacle. This exhibits interesting behavior and reveals some problems due to the mesh dependency of the dynamics of the problem. Physical constraints are also of concern: In particular the corrections are not guaranteed to be divergence-free. In 2D one can predict a streamfunction instead of a velocity. A comparison between velocity and streamfunction prediction will provide some insight to physics informed machine learning.

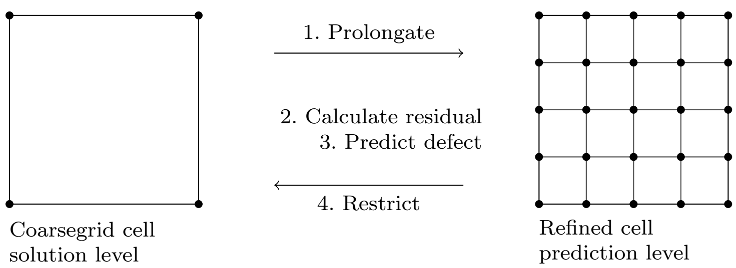

Figure 1: Idea of the presented method. On the refined level a residual is calculated, which is then used as the input for a neural network predicting the defect

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