Data driven approximation of parametrized PDEs by Reduced Basis and Neural Networks

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We are interested in the approximation of partial differential equations with a data-driven approach based on the reduced basis method and machine learning. We suppose that the phenomenon of interest can be modeled by a parametrized partial differential equation, but that the value of the physical parameters is unknown or difficult to be directly measured. Our method allows to estimate fields of interest, for instance temperature of a sample of material or velocity of a fluid, given data at a handful of points in the domain. We propose to accomplish this task with a neural network embedding a reduced basis solver as exotic activation function in the last layer. The reduced basis solver accounts for the underlying physical phenomenonon and it is constructed from snapshots obtained from randomly selected values of the physical parameters during an expensive offline phase. The same full order solutions are then employed for the training of the neural network. As a matter of fact, the chosen architecture resembles an asymmetric autoencoder in which the decoder is the reduced basis solver and as such it does not contain trainable parameters. The resulting latent space of our autoencoder includes parameter-dependent quantities feeding the reduced basis solver, which – depending on the considered partial differential equation – are the values of the physical parameters themselves or the affine decomposition coefficients of the differential operators.

References

 Dal Santo, Deparis, Pegolotti. Data driven approximation of parametrized PDEs by Reduced Basis and Neural Networks arXiv:2018