An Artificial Neural Network based approach to Model Order Reduction of time-dependent models: application to multiscale cardiac simulation

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The numerical solution of complex time-dependent mathematical models, in the form of PDEs or ODEs, may lead to an unbearable demand for computational resources, especially when there is the need to perform simulations multiple times (multi-query) and for many different inputs, or when the results must be provided in nearly real-time. This motivates the development of reduced models, that is computationally tractable, lower dimensional mathematical models, yet reproducing the results of the high-fidelity (HF) model. Approaches to Model Order Reduction (MOR) can be split in two categories [1, 2, 4]: model-based approaches, which exploit the knowledge of the HF model, and data-driven approaches, which build the reduced model upon a collection of input-output pairs, from which the dynamics of the system is inferred. We here propose a data-driven MOR approach, based on Artificial Neural Networks (ANNs), applicable to linear and nonlinear time-dependent systems of ODEs or PDEs. We set the problem in an abstract form, where we look for the maximum-likelihood estimation of the HF model into a class of simpler models, thus yielding an optimization problem, where the the model itself is the unknown. Then, we select a class of reduced models, written in as a system of ODEs, whose right-hand side is represented by an ANN, which we train to learn from input-output pairs obtained from the HF model. This approach also allows to incorporate into the supervised learning process available a priori information on the nature of the model. Moreover, unlike most projection-based MOR methods, the proposed approach does not require modifications nor special adaptations when the HF model is nonlinear or features non-affine parametric dependence [4].

We exploit the reduced model developed so far in the context of multiscale cardiac simulations [3], specifically for building a surrogate model for active force generation in cells of the cardiac muscle. Indeed, contraction and relaxation of the heart follows from the concerted action of several core models, namely electrophysiology, force generation at the cellular level, macroscopic active and passive mechanics, and fluid dynamics. These models span a very wide range of spatio–temporal scales, which numerical methods should be able to capture in a computationally efficient and accurate manner. Force generation models in particular are extremely complex and may literally require solving thousands of ODEs just to determine the force generated in a single muscular cell [5]; as a matter of fact, spatial and temporal scales herein involved are the smallest ones in the integrated heart model. In this work, we replace the active force model in the integrated heart model with our data-driven MOR exploiting ANNs. We show that how approach yields very accurate results and at the same time allows a dramatic reduction of computational costs.

References

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